MTH 320: HW 2

Instructor: Matthew Cha

Due: January 23, 2019

Problems from Abbott's book (2nd ed.) are labeled by **Abbott chpt.sec.#.**

1. **(Abbott 1.4.2)** Let $A \subset \mathbb{R}$ be non-empty and bounded above, and let $s \in \mathbb{R}$ have the property that for all $n \in \mathbb{N}$, $s + \frac{1}{n}$ is an upper bound for A and $s - \frac{1}{n}$ is not an upper bound for A. Show $s = \sup A$.

2. **(Abbott 1.4.3)** Prove that $\bigcap_{n=1}^{\infty}(0, 1/n) = \emptyset$.

3. **(Abbott 1.4.4)** Let $a < b$ be real numbers and consider the set $T = \mathbb{Q} \cap [a, b]$. Show that $\sup T = b$.

4. **(Abbott 1.4.8(a)(d))**

   (a) Give an example of two sets $A, B \subset \mathbb{R}$ with $A \cap B = \emptyset$, $\sup A = \sup B$, $\sup A \notin A$ and $\sup B \notin B$.

   (d) Prove or disprove the following: There exists a sequence of closed bounded (not necessarily nested) intervals $I_1, I_2, I_3, \ldots$ with the property that $\bigcap_{n=1}^{N} I_n \neq \emptyset$ for all $N \in \mathbb{N}$, but $\bigcap_{n=1}^{\infty} I_n = \emptyset$.

5. **Abbott 1.5.3(c)** Prove that if A_n is a countable set for each $n \in \mathbb{N}$ then $\bigcup_{n=1}^{\infty} A_n$ is countable. (See the book for a hint.)

6. **(Abbott 1.5.5)** This problem shows that $\sim$ is an equivalence relation.

   (a) Show that $A \sim A$ for every set A.

   (b) Show that if $A \sim B$ show that $B \sim A$.

   (c) Show that if $A \sim B$ and $B \sim C$ show that $A \sim C$.

7. **(Abbott 1.5.8)** Let B be a set of positive real numbers with the property that adding together any finite subset of elements from B always gives a sum of 2 or less. Show that B must be finite or countable.

8. **(Abbott 1.6.10(a)(b))**

   (a) Is the set of all functions from $\{0, 1\}$ to $\mathbb{N}$ countable or uncountable?

   (b) Is the set of all functions from $\mathbb{N}$ to $\{0, 1\}$ countable or uncountable?